

## **REGRESSION ANALYSIS OF COUPLING EFFECTS OF THERMAL AND EXTERNAL FORCES ON A PRECISION MACHINE CENTER**

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### **ABSTRACT**

The thermal, gravity, and external load in the upward direction on the spindle of a machine center and their coupling effects are studied in the paper. The finite element method is used to calculate the variety of combination of load-displacement and load-stress results, then regression algorithm is used to get corresponding equations. As long as the regression equations for certain kinds of machine center are obtained, then substitute temperature and load into these equations, the result will be obtained. No tedious finite element calculation should be done repeatedly, so that a tremendous amount of time will be saved. To shorten the developing time is to reduce cost and increase competition. Four cases are studied: (1) External load on the spindle without considering temperature rising and gravity force; (2) External load on spindle as well as gravity force on machine center, but no temperature rising; (3) Temperature rising at the spindle with gravity force; (4) Temperature rising, gravity force and external load on the spindle coupling together. The corresponding regression equations are recommended for the future use. The regression surfaces are also given for the coupling of temperature and loading case to give designers a guidance in their machine center design.

**KEYWORDS:** Machine Center, Regression Analysis, Coupling Effects

### **INTRODUCTION**

The purpose of this paper is to propose a method to use equations and diagrams to envelope the maximum displacement and von Mises stress for a machine center under different combination of external loadings. A full range of the coupling effects of thermal and other external forces, such as gravity, and working loads on the precision CNC machine center are given, so that during design, the developing engineers can just substitute load and temperature into the proposed equations, then stress and deformation can be obtained immediately. No tedious finite element analysis procedures have to be done repeatedly to get results. A tremendous amount of time can be saved, in other words, money can be saved. Taiwan is one of the well-known places for machine tools manufacturing in Asia. In Asia, only a few countries such as Taiwan, Japan, South Korea, and China are equipped with the capability of making machine tools. Because of the flexibility and high-quality, the whole world becomes the market for machine tools made in Taiwan. The CNC machine center is the most important exporting machine for Taiwan, and it accounts for about 35% of the total machine tools exporting value [5]. How to shorten a new machine developing period, and find out the weak points before issuing engineering design diagrams for manufacturing are critical in this competitive world. To shorten the developing period is to reduce cost, and the very purpose in this study may fulfil the need for machine center manufacturers.

The author uses the finite element method along with one of the most popular commercialized softwares ANSYS Workbench to analyze the stresses and deformations of the whole machine center under a variety of external loading conditions (normal working, gravity, and thermal). Their coupling effects, such as external loading interact with gravity

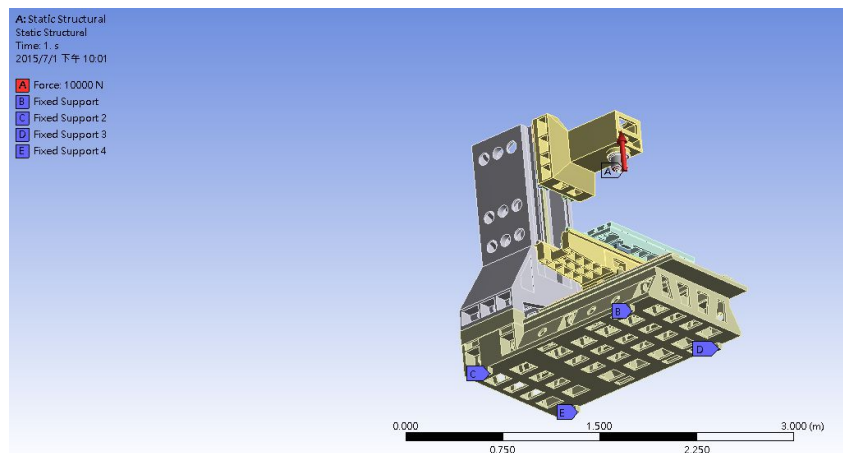
and thermal load, which will even complicate the analysis procedure. In this study, regression equations, which cover the whole range of loadings and temperatures for normal working condition of a machine center, will be given to shorten the design time.

**ANALYSIS PROCEDURES**

This study uses Autodesk Inventor to build and design a 3D model of the machine center. To avoid making the discretized solid model divergent during analysis, one has to simplify the model to eliminate all the large curvatures of the original model [1]. The simplified model then is transferred to a parasolid file, and imported to ANSYS Workbench for the finite element analysis. The stress, deformation, and post processing of the results are also dealt by means of ANSYS Workbench.

**Boundary Condition and Material Properties**

The boundary condition is assumed to fix on four locations at the base of the machine. This is a reasonable assumption since the machine should be fixed on the floor for avoiding any movement during working. The boundary condition is shown in Figure 1, and material properties are given in Table 1.



**Figure 1: Fixed Boundary Condition and External Force on Positive Spindle Direction (+Z)**

**Table 1: Average Mechanical Properties of Typical Engineering Materials (SI Unit) [2,3]**

Material	Density $\rho$ ( $kg / m^3$ )	Modulus of Elasticity $E$ ( $Gpa$ )	Modulus of Rigidity $G$ ( $Gpa$ )	Yield Strength $\sigma_y$ ( $Mpa$ )	Poisson's Ratio $\nu$	Coefficient of Thermal Expansion $\alpha(10^{-6}) / ^\circ C$
FC-30	7,250	123.0	48.8	227.5	0.26	11.0

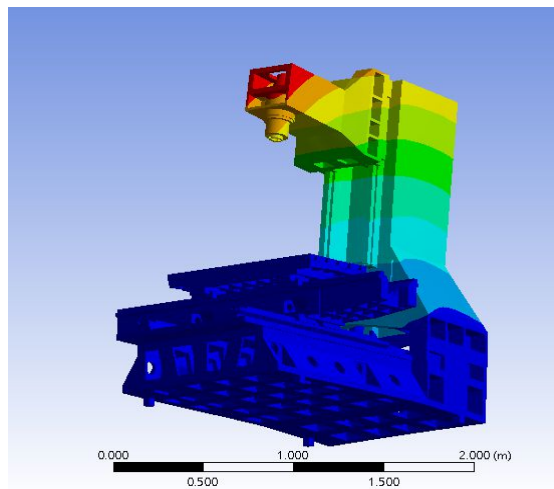
**External Loadings**

Four cases are used in the static analyses. The first case simulates the spindle of machine center during working. A variety of external upward loadings from 100N to 10,000N are subjected to the positive Z axis of the spindle (Figure 1), and their corresponding maximum deformations and von Mises stresses are calculated. The second case considers the combination effect of external upward spindle loading and gravity force. This is a more realistic analysis because gravity force always subjects on the machine. The third case discusses the thermal effect and gravity force together. The fourth

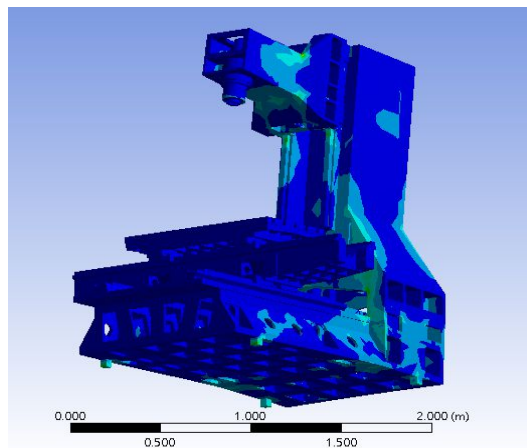
case couples upward spindle load, gravity force, and temperature effect together to simulate the real condition of machine center during working. Only the elastic analysis is used, and the plastic condition is not considered here.

### External Loading on Spindle (+Z Direction) with and without Considering Gravity Force

For simulating the working condition of the machine, one assumes the loading is on the positive Z axis (+Z) of the spindle. The real loading on the spindle depends on the cutting depth, rotating speed, temperature, and material of the object which one is working on, hence, one of the realistic ways is to let the external loading vary in a range. In this study the author assumes that the external loading varies from 100 N to 10,000 N. This range can cover almost the working conditions in the machine life. In this subsection both external loading with and without gravity force ( $9.81\text{m/s}^2$ ) are analyzed. Figure 2 and 3 are the results of displacement (deformation) and von Mises stress under +Z spindle load, which varies from 100N to 10,000N. They are just a demonstration, not all figures are included to save space.



**Figure 2: Deformation of the Machine under 10,000 N in Positive Spindle Direction (Without Gravity Force)**



**Figure 3: The von Mises Stress of the Machine under 10,000 N in Positive Spindle Direction (Without Gravity Force)**

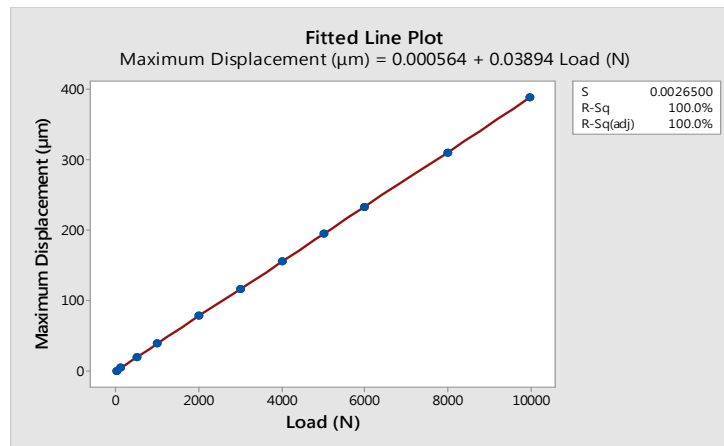
The summary of the analysis results of upward loading on the spindle with and without consideration of gravity force are shown in the following table.

Table 2: External loading on the upward direction of spindle and the maximum displacement ( $10^{-6}$ m) as well as von Mises stress (kpa)

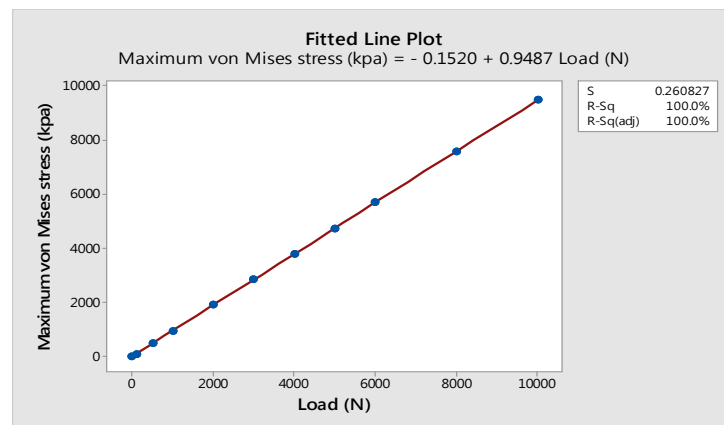
Load (N)	Maximum Displacement ( $\mu\text{m}$ ) (No Gravity)	Maximum von Mises Stress (kpa) (No Gravity)	Maximum Deformation ( $\mu\text{m}$ ) (with Gravity)	Maximum von Mises Stress (kpa) (with Gravity)
0	0.0	0.0	108.1	6303.0
100	3.9	94.9	104.2	6230.7
500	19.5	473.5	88.6	5941.3
1000	38.9	948.7	69.2	5580.0
2000	77.9	1897.4	31.4	5180.0
3000	116.8	2846.1	18.4	4814.9
4000	155.8	3794.8	48.0	4455.8
5000	194.7	4743.5	86.9	4110.9
6000	233.7	5692.2	125.8	4782.3
8000	311.5	7589.6	203.6	6679.7
10000	389.4	9486.9	281.5	8677.1

**Regression of Results without Gravity Force**

The regression of load-displacement and load-stress relationships are shown in Figure 3 and Figure 4.



**Figure 3: Regression Equation of Deformation and External Load (No Gravity Force)**



**Figure 4: The Regression Equation of von Mises Stress and External Load (No Gravity Force)**

The R-Sq(adj) at the legend of the figure above is the adjusted coefficient of determination. It measures the proportion of variability in the response to be explained by the regression equation [4]. R-Sq(adj) 100% in the figures above means the regression equation can 100% explain both displacement and von Mises stress with respect to the external load.

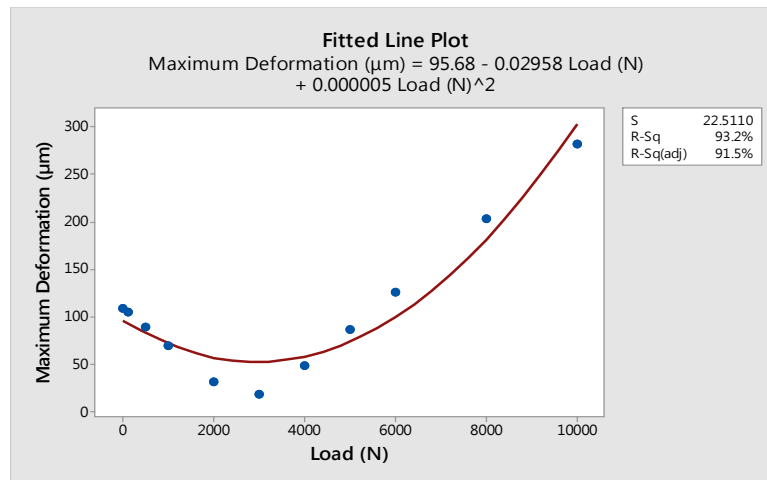
The load-displacement regression equation is as follows (no gravity force):

$$D(\mu m) = 0.000564 + 0.03894Load(N) \tag{1}$$

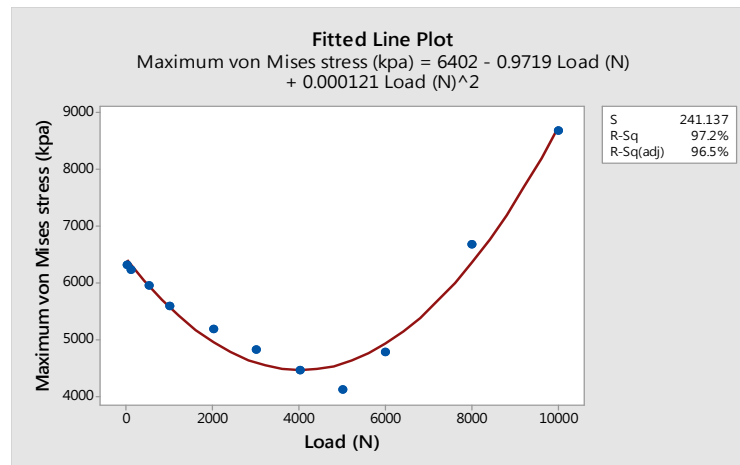
The load-stress regression equation is as follows (no gravity force):

$$Stress(kpa) = -0.152 + 0.9487Load(N) \tag{2}$$

**Regression of the Results with Gravity Force**



**Figure 5: Load-Displacement Regression Equation (No Thermal but with Gravity Force)**



**Figure 6: Load-Stress Regression Equation (No Thermal but with Gravity Force)**

The second order regression equation can explain load-displacement and load-stress relationship up to 91.5% and 96.5% respectively. Surprisingly, the relationship of both load-displacement and load-stress are not linear. Instead, a

quadratic regression can fit the relationship quite well. Because the external loading on the spindle and gravity force interact together, the maximum displacement and stress may not occur at the same location as in the case without considering gravity force.

The load-displacement regression equation is as follows (with gravity force):

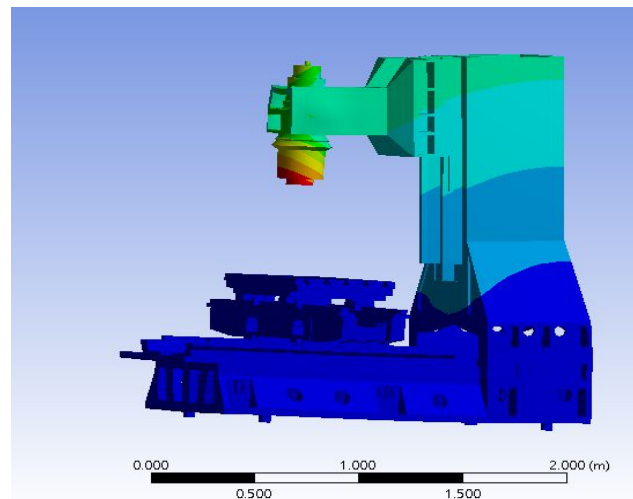
$$D(\mu m) = 95.65 - 0.02958Load(N) + 5 * 10^{-6} Load^2(N) \tag{3}$$

The load-stress regression equation is as follows (with gravity force):

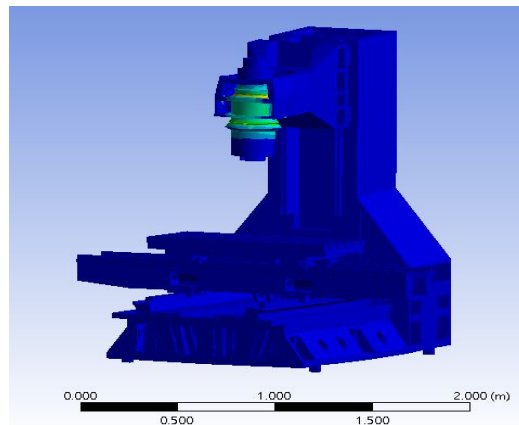
$$Stress(kpa) = 6402 - 0.9719Load(N) + 1.21 * 10^{-4} Load^2(N) \tag{4}$$

**Temperature Increases in the Spindle of Machine Center with Gravity Force**

The temperature of the spindle will rise during working. The environmental (reference) temperature is set at 22 °C. The temperature of the spindle is assumed to rises uniformly to the temperature one set. Say, if one sets the spindle temperature at 30 °C, that means the whole spindle’s temperature is 30 °C. The temperature is assumed to rise from 30°C to 100 °C during working. The results are shown in Table 3.



**Figure 7: Deformation of Machine Center with Spindle Temperature at 100 °C (Wih Gravity Force)**



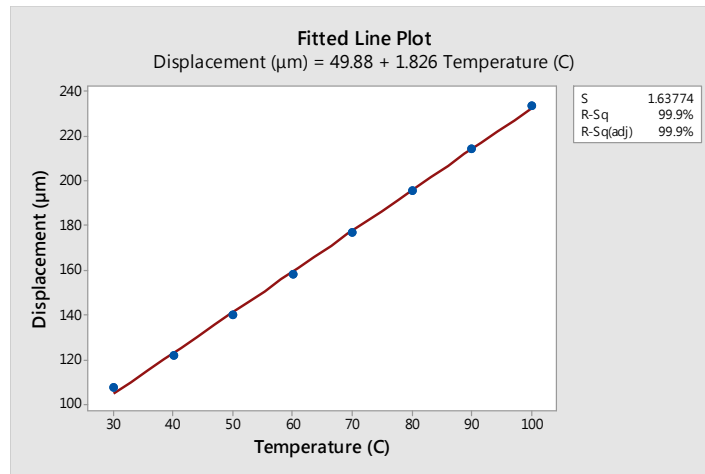
**Figure 8: The von Mises Stress of Machine Center with Spindle Temperature 100 °C (With Gravity Force)**

**Table 3: Temperatures of Spindle and their Effect on Machine’s von Mises Stress and Displacement**

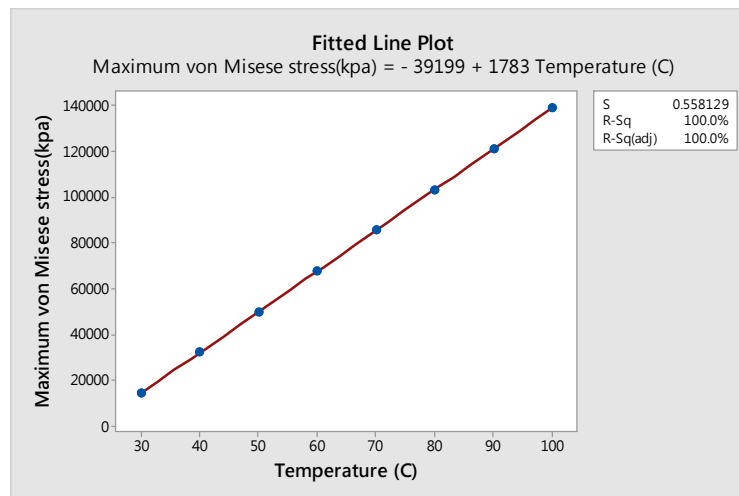
Temperature (°C)	von Mises Stress (10 <sup>6</sup> N/m <sup>2</sup> , Mpa)	Displacement ( μ m = 10 <sup>-6</sup> m)
30.0	14.29	107.66
40.0	32.12	121.6
50.0	49.95	139.85
60.0	67.78	158.14
70.0	85.61	176.98
80.0	103.44	195.83
90.0	121.27	214.69
100.0	139.10	233.55

**Regression Equation of Temperature Rising on the Spindle**

The regression equation for temperature-displacement relationship is shown in Figure 9, and temperature-stress relationship is shown in Figure 10.



**Figure 9: Temperature - Deformation Regression Equation (With Gravity Force)**



**Figure 10: Temperature - von Mises Stress Regression Equation (With Gravity Force)**

The linear regression equation can explain load-displacement and load-stress relationship almost up to 100%.

The load-displacement and load-stress regression equations are as follows (with gravity force):

$$D(\mu m) = 49.88 + 1.826 \text{Temperature} (^{\circ} C) \quad (5)$$

$$\text{Stress}(kpa) = -39199 + 1783 \text{Temperature} (^{\circ} C) \quad (6)$$

### Coupling Thermal, Loading on Spindle, and Gravity Force on the Machine Center

In this subsection, the author tries to couple thermal, gravity, and external loading on the spindle together to simulate a real condition of a machine center during work.

### Coupling Effects of Displacement on the Machine Center

After analyzing several cases, the maximum displacement of the machine center can be summarized in Table 4.

**Table 4: The Maximum Displacement of the Machine with Coupling Effects of Thermal and External Load**

Temperature ( $^{\circ}C$ ) \backslash Loading (N)	0	2000	4000	6000	8000	10000
22	108.07	31.44	48.04	125.76	203.61	281.48
40	121.60	61.66	82.61	152.74	225.15	298.17
60	158.14	99.03	129.88	196.53	266.55	337.89
80	195.83	136.63	178.16	242.45	310.64	380.63
100	233.55	175.01	226.92	289.42	356.05	424.79

### Coupling Effects of von Mises Stress on the Machine Center

After analyzing several cases, the maximum stress of the machine center can be summarized in Table 5.

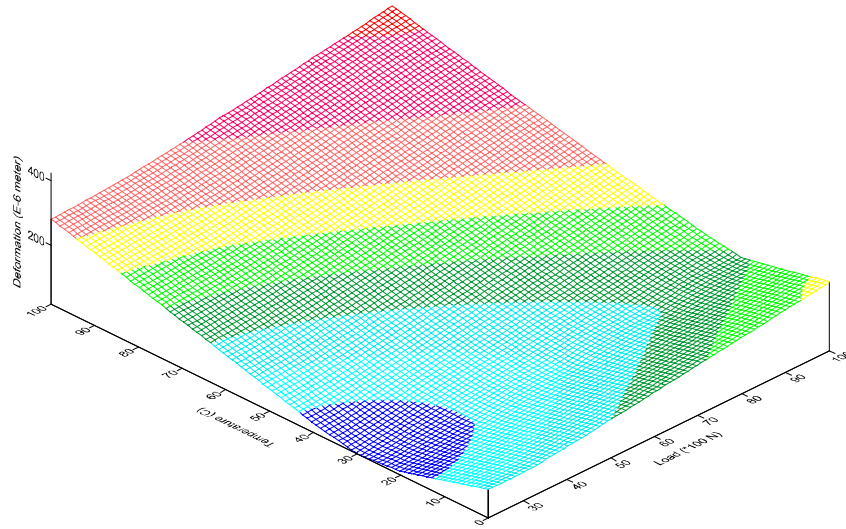
**Table 5: The Maximum von Mises Stress of the Machine with the Coupling Effects of Thermal, Gravity and Upward Spindle Load (+Z) (unit =  $10^6 N/m^2$ , Mpa)**

Temperature ( $^{\circ}C$ ) \backslash Loading (N)	0	2000	4000	6000	8000	10000
22	6.30	5.18	4.46	4.78	6.68	8.58
40	32.12	32.07	32.02	31.96	31.91	31.86
60	67.78	67.73	67.67	67.62	67.57	67.52
80	103.44	103.39	103.33	103.28	103.23	103.18
100	139.10	139.04	138.99	138.94	138.89	138.83

The regression equation of deformation with respect to temperature and external load is:

$$D(\mu m) = -1.3 + 1.98 \text{Temperature} (^{\circ} C) + 0.0179 \text{Load} (N) \quad (7)$$

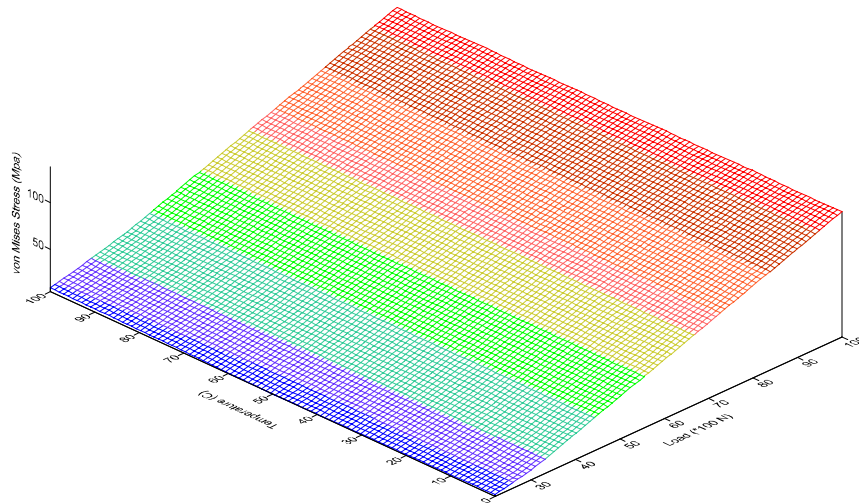




**Figure 11: The Regression Surface of the Temperature and External Load on the Deformation of the Machine**

The regression equation of temperature, external force to the von Mises stress can be expressed and plotted as:

$$Stress(Mpa) = -33.9 + 1.72Temperature(^{\circ}C) + 5.8 * 10^{-6} Load(N) \tag{8}$$



**Figure 12: The Regression Surface of Temperature and External Load on the von Mises Stress of the Machine**

Note that the regression surface of either von Mises stress or deformation with the coupling effects of temperature and external load is the record of the maximum values. They do not always occur at the same place. Take stress as an example, if considering gravity force, when the external load is small, the maximum stress is at the base of machine, and when the external force increases, the maximum stress is at the connection of head and column. The explanation of the deformation is identical to that of stress, only the maximum deformation is traced, and may not occur at the same location.

**CONCLUSIONS**

After performing static analyses, the following regression equations can be obtained. In the analysis, the author already considered a wide range of loadings for all possible working conditions of the machine center. The research and

developing engineers just need to find out the corresponding equations and substitute with temperature and external loading values, the deformation and stress can be obtained immediately. No tedious finite element analysis is necessary. The regression equations for different cases are:

- Regression equations of displacement and stress with external load on +Z direction at the spindle (no thermal and no gravity effects) are:

$$D(\mu m) = 0.000564 + 0.03894 * Load(N)$$

$$Stress(kpa) = -0.152 + 0.9487 Load(N)$$

- Regression equations of displacement and stress of machine with external load on +Z direction at the spindle and gravity force (no thermal effect)

$$D(\mu m) = 95.65 - 0.02958 Load(N) + 5 * 10^{-6} Load^2(N)$$

$$Stress(kpa) = 6402 - 0.9719 Load(N) + 1.21 * 10^{-4} Load^2(N)$$

- Regression equations of displacement and stress with only temperature rises at the spindle (no external load but with gravity force)

$$D(\mu m) = 49.88 + 1.826 Temperature(^{\circ}C)$$

$$Stress(kpa) = -39199 + 1783 Temperature(^{\circ}C)$$

- Regression equations of coupling effects of temperature, gravity force and positive (+Z) spindle load:

$$D(\mu m) = -1.3 + 1.98 Temperature(^{\circ}C) + 0.0179 Load(N)$$

$$Stress(Mpa) = -33.9 + 1.72 Temperature(^{\circ}C) + 5.8 * 10^{-6} Load(N)$$

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