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NONLINEAR DYNAMICS, CHAOS & APPLICATIONS TO TUNABLE LASERS & RELATED AREAS

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ABSTRACT

This article looks at the applications of the ideas of nonlinear dynamics and chaos from applied mathematics and other ideas from pure mathematics (outlined elsewhere) towards realization of chaotic systems and their applications to design of tunable circuits with relevance to electronic circuit design and related areas. Some applications to medicine, biology and health care are also examined.

KEYWORDS: Feedback, Difference Map, Bi-Stability, Multi-Stability, Deterministic, Nonlinear, Acousto-Optic, Acousto-Optic Cell, Phased Array, Chaos, Tunable Lasers, Period=Doubling, Intermittency, Hysteresis, Stem Cells, Quantum Computing, Phase Space..Chaotic Search..Etc

INTRODUCTION

As is well known, feedback in electronics circuits can endow the system with rich and interesting behavior, including deterministic chaos (with its properties of combining sensitivity to initial conditions and boundedness etc)

Difference equations / discrete maps are the first and simplest examples of dynamical systems that have been studied and examined.

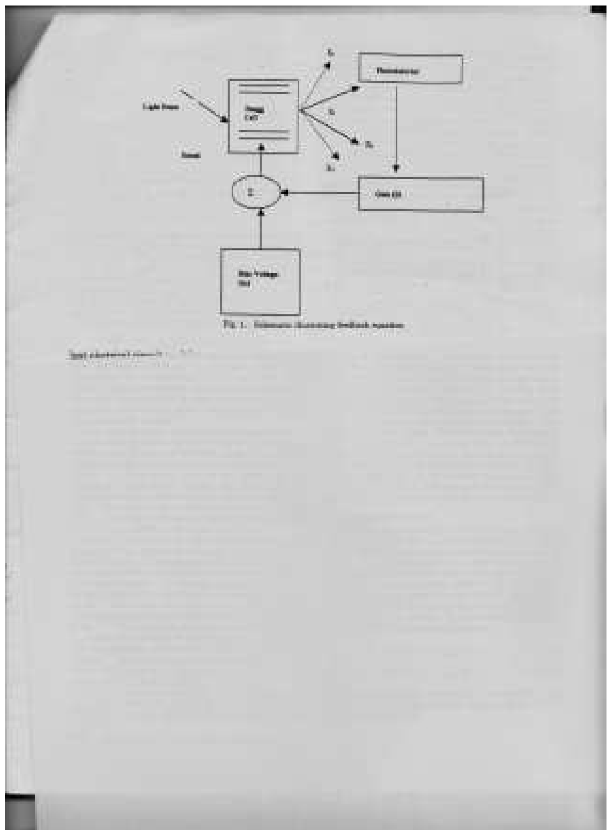
For example, the logistic equation $x_{n+1} = r*x_n*(1-x_n)$ is an example of a map that shows a variety of behaviors for various values of the gain parameter : r. For low values of r, the system settles down to a stable fixed point (zero) while for higher values of r, it shows multi-stable behavior where the attractor / asymptotic / limiting behavior of the system would depend on the initial value, x_0. For even larger values of r, the system displays the characteristic sequence of bifurcations with period-doubling cascades characterized by the system cycling between the various points of period n. As the gain parameter is increased, the system enters the chaotic regime with periodic windows.

Reference 1 along with the references mentioned therein seeks to extend and generalize the above to the optical domain where they seek to realize the optical equivalent of bistability, multistability, periodic cycles and chaos (with the possibility of periodic windows). The references mentioned therein also try to realize the optical equivalent of a flip-flop using the concept of optical bistability / multistability

Essentially, they investigate the dynamics resulting in the operation of an acousto-optic piezoelectric cell in the so-called Bragg regime (hence called a piezo-electric or acousto-optic bragg cell) which is driven by an RF signal which makes the cell the equivalent of a phase grating with periodic modulation of the refractive index with a period depending on the frequency of the RF driver) where the light diffracted / scattered by the cell (or rather the light scattered in one or some of the directions) is detected and fed back with after amplification by a gain factor and added to an RF signal acting as bias. Essentially, the AO-cell acts as the equivalent of a function generator.

METHODS (Refs 1 & 2)

The dynamics can be conceptually captured by the following schematic:



In the above, there is also an implicit phase delay parameter with the various denoting the various orders of light scattered in various directions.

Essentially, the essence of the behavior of the above system can be captured in the following systems of equations:

$$I_1(n+1) = I_{inc} f(\alpha_{n+1})$$

$$\alpha_{n+1} = \alpha_0 + \beta I_1(n)$$

Here n and n+1 denote the instants of time which in the case of I_1's or I_n's generally, is not subscripted to avoid confusion with the different orders / beams where the function f at any instant is determined by solving the following system of differential equations (4 in our case) :

$$\frac{dE_{-1}}{d\xi} = -j\,\frac{\alpha\ \exp(-jQ\xi)\,E_0}{2},$$

$$\frac{dE_0}{d\xi} = -j\,\frac{\alpha[\exp(jQ\xi)\,E_{-1} + E_1]}{2},$$

$$\frac{dE_1}{d\xi} = -j\,\frac{\alpha[\exp(jQ\xi)\,E_2 + E_0]}{2},$$

$$\frac{dE_2}{d\xi} = -j\,\frac{\alpha\ \exp(-jQ\xi)\,E_1}{2},$$

Where E_n (n=1...4 in our case) are the amplitudes of the four orders,

α is the peak phase delay , α_0 corresponds to the phase delay corresponding to the bias voltage,

Q denotes the Klein-cook parameter (can be related to effective thickness / length of the cell)

and

ξ denotes the normalized distance in the direction of propagation. (In a generalized scenario, one may need 2 variables for the 2 directions)

and the boundary conditions at ξ = 0 are :

$E_{-1} = E_1 = E_2 = 0$ and

$E_0 = 1$ (in terms of the incident amplitude E_{inc})

β denotes the feedback gain parameter

The I_n's denote the intensities of the E_n's

The system goes on to examine the dynamics of the system as a function of the above parameters , i.e.

1. the gain β,
2. the bias voltage α_0 or rather the peak phase delay corresponding to it

3. the value of the actual peak phase delay α

4. The value of the effective "thickness" (possibly length) - Q

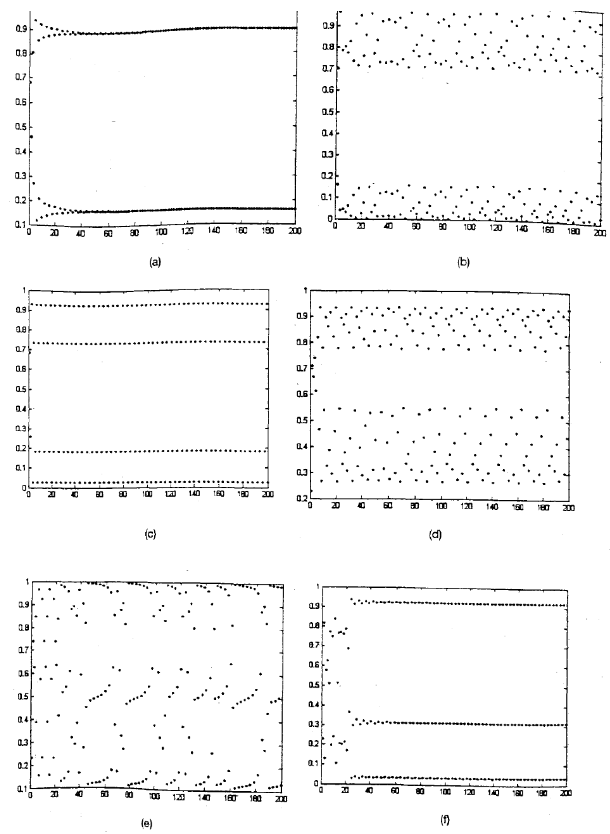
5. Table 1 below summarizes the highlights of the dynamical behavior of the system defined above

Table 1. Summary of Dynamics of I_1 When Bias Voltage is Not 0

Gain	Bias Voltage (α_0)	Dynamic Behavior for High Q	Dynamic Behavior for Low Q
2.41	2	Monostable/bistable (Fixed Point) (Fig. 8.1 in Ref. 17)	Monostable/bistable (Fixed Point)
2.8	2	Period two (Fig. 8.2 in Ref. 17)	Monostable/bistable (Fixed Point)
3.2	2	Period two	Period two
4.2	2	Period four (Fig. 8.5 in Ref. 17)	Period two (Fig. 2(a)]
4.5	2	(The high Q behavior needs looking into)	Period four
5	2	Chaos/irregular [Fig. 2(b)]	Period four [Fig. 2(c)]
8	2	Chaos/irregular	Chaos/irregular
2.41	3	Period two	Monostable/bistable (Fixed Point)
4.4	3	Period four	Period two
4.7	1	Chaos/irregular ? (less irregular than low Q) [Fig. 2(e)]	Chaos/irregular [Fig. 2(d)]
6	1	Most probably, chaotic or irregular	Period three [Fig. 2(f)]
5	4	Not examined[a]	Period two [Fig. 2(g)]

[a]Because the hysteretic plots for high Q in Section 5 did not show any significant stable behavior for any value of bias voltage within the range examined (-1 to 7).

The following figures depict the significant features of the dynamics in more detail (Refs 1 & 2) including the period 3 behavior mentioned above which occurs well into the chaotic regime



2. (a) Plot of I_1 vs. n for $Q = 3\pi$, $\alpha_0 = 2$, $\beta = 4.2$; (b) $Q = 8\pi$, $\alpha_0 = 2$, $\beta = 5$ (onset of chaos for high Q); (c) $Q = 3\pi$, $\alpha_0 = 2$, $\beta = 5$ haos, still at period 4); (d) $Q = 3\pi$, $\alpha_0 = 1$, $\beta = 4.7$ (chaos sets in already for low Q); (e) $Q = 8\pi$, $\alpha_0 = 1$, $\beta = 4.7$); (f) $Q = 3\pi$, $\alpha_0 =$ $= 6$ (low Q period 3 cycle within the chaotic regime. The transients show chaotic behavior, which reinforces our earlier statement ; period three being a periodic window within chaos); (g) (on the next page) $Q = 3\pi$, $\alpha_0 = 4$, $\beta = 5$ (period 2 at high β).

The figures corresponding to the examination of the hysteretic behavior of the system (Refs 1 & 2) have not included here. The above also would possibly take into account the parameter corresponding to the phase / time delay due to the feedback

ADDITIONAL NOTES

The above is an example of a hybrid dynamical system where the differential equations correspond to the analog system and the difference map relating to the discrete feedback system,. One additional possibility is to extend the above difference map to a completely analog system (possibility after incorporating the time delay due to feedback through phase delay parameter). This could also be extended to internal feedback through confinement etc.

RESULTS / MAJOR APPLICATIONS / IDEAS

As mentioned above, Rich dynamics are observed including bistability, multistability and associated hysteretic behavior, period-doubling cascades / bifurcations, the onset of chaos for certain values of the parameters and periodic windows within the chaotic regime (including period 3 behavior). The essence of the cyclic behavior seems to be the delicate interplay between the instability of the individual fixed points within the cycle and the stability of the cycle as a whole. The periodic windows are also characterized by intermittency or bursting behavior where the periodic behavior is interspersed by irregular / chaotic behavior. It would also be interesting to extend the above to coherent feedback, i.e. involving the amplitudes rather than the intensities.

An interesting idea in this context could be to apply the dynamics resulting from the composition of functions in the scheme above towards the realization of tunable oscillators / lasers etc. This would have the added advantage of making the oscillator frequency independent of / or nonlinearly *dependent* on the input voltage. This also seems to be linked to generating clock pulses of appropriate frequencies through the harnessing of chaos / period-doubling / periodic windows etc which could have applications in design of variable rate clocks etc which could find applications in handling variable rate traffic. This could also be used for rapid tuning by pushing the system to the chaotic regime and then controlling the chaos by very minute variations of the gain etc so that the system settles down to a stable periodic cycle within a nearby window. This could also be used for designing counters modulo n where n is tunable / variable as in frequency division / multiplication circuits using flip-flops but with minimal complexity in terms of gates etc. The embedding of the periodic cycles in chaos which is characteristic of intermittency could be key to the design of complex pulse width and other complex modulation schemes possibly characterized by space-filling graphs. Another advantage of the chaotic / cyclic behavior could be the realization of circuits that are robust in the presence of noise (links to the attractor idea) as also realization of resilience against interference through the sensitivity to initial conditions that is characteristic of chaos and that reduces the possibility of cross talk.

It is also interesting to speculate if the chaotic dynamics of the above could be used to realize quantum computing etc, especially in the sense that classical measurement (that possibly corresponds to rounding off and cut-offs in digital circuits possibly corresponding to so called "Q**uantization Noise**") is delayed till the end of the computation. At a practical level, this possibly avoids accumulation of rounding off errors. This seems to be one example of the efficiency of so called "lazy" evaluation.

The independence or nonlinear dependence of frequency on input voltage also has the advantage of reducing power consumption and hence heat dissipation which along the reduction in complexity could go a long way in reducing the footprint of oscillators, clocks, lasers and related devices.

The above is also linked to the theme of relaxation oscillations that are typically characterized by 2 or more time scales that are fairly widely separated.

The period n cycle could also be key to the design of n-ary logical elements which could be useful for squeezing more bandwidth in terms of bit rate for communication lines with a given baud rate.

At a more abstract level, one could extend the above to the idea of the system cycling between points in complex physical / frequency or phase space or even points in a Hilbert space of functions.

The above mentioned ideas also seem to usher in new paradigms of optical computing in addition to that of optical image processing.

The above may also be relevant to the design of lasers with a period of the order of femto-seconds or lower

The reference (along with Reference 2 and some other references cited in both Ref 1 and 2) also tries to extend the above to chaos in the direction of the beam. One way to realize this seems to be generate chaotic RF signals that drive the AO cell.

Reference 3 also talks about the use of combination of RF signals with incommensurate frequencies to drive the AO cell and examines the pattern of diffraction of the beams(s)

An interesting idea in this context could be to realize a kind of inversionless lasing in an AO cell using the interplay between symmetry, entropy, free energy and stability. This could also be extended to examine the creation of confined and traveling modes created by the coupling of electro-magnetic and acoustic waves *(acousto-optic solitons)*

One of the major applications of the above ideas is in the optical entrainment of biological and biochemical clocks by tuning the laser to pulse at a rate that is synchronous or in resonance with specific biological / biochemical reactions that are to be selectively activated / reset.

. Another application in this context is that of utilizing the above ideas (especially the idea of directional chaos and / or phase locking) to destroy tumors / cancer cells / antigens where the shape and / or anisotropy of the targets which could be possibly linked to the frequency of modes of vibrations and hence to the spectral absorbances of these cells (links to Fluorescence Resonance Energy Transfer – FRET imaging etc). In this context, it would be interesting to extend the idea of directional chaos to chaotic search / tracking in various directions in function / phase space (and other abstract spaces as mentioned earlier) for example in the space of temporal pulse and spatial beam profiles, pulsing frequencies, wave numbers etc.

Conversely, cancer cells etc can be induced to transform themselves to stem cells either directly through input of metabolite / biochemicals or indirectly through photo catalysis of the appropriate biochemical reactions at the appropriate rates with the electromagnetic radiation acting as a forcing / entraining parameter. For example, photo catalysis / entrainment of reactions involving cyclins and cyclin-dependent protein kinases (Cdk) (that are involved in the cell division cycle (and possibly including hormones etc)) involving activation, repression , inhibition etc. Stem cells can also be induced to differentiate themselves into specific cell types through the above mechanisms. One interesting theme is the link between differentiation of stem cells and themes from quantum mechanics such as the theme of "measurement/wave function collapse", especially if we look at the stem cells as being in a superposition state. It is interesting that that the growth of cancer cells etc could possibly involve bursting behavior which is also a characteristic of intermittency. The above ideas could also have applications in wound healing, tissue regeneration etc. It would be interesting to extend this idea to computing devices that "grow", "develop" and "learn".

Again, in the biological context, it would be interesting to examine the application of the ideas outlined above not only to realize and implement artificial neural networks but also to study the workings of the human brain / mind and the major paradigms involved therein., including optical and chaotic neural networks. When implemented optically, the above may also be used to study human vision.

The above ideas related to intermittent chaos with embedded periodic cycles may also be relevant to the study design of adaptive systems, for example in biology. Research has been conducted on the advantages of chaos, especially in the context of intermittency, for the functioning of biological systems both at the organ level and at the physiological system level such as heart, brain, immune, circulatory system, respiratory system etc. This may also be relevant to the design of pacemakers, defibrillators and other artificial devices (such as heart valves, heart lung machines etc) that may be applicable in a range of disorders including heart disease, diabetes and other metabolic disorders, asthma, epilepsy, neural degenerative disorders such as Parkinson's disease, palsies etc (especially since the latter are known to be characterized by repetitive rhythms / oscillations). It would also be interesting to study of the coupling of the systems to each other and to external entraining forces such as electromagnetic waves that drive

Biological clocks. As in a laser, feedback may also be relevant to harnessing energy and thus minimizing external power requirements. One interesting theme relates to the application of the above ideas of intermittency to study turbulence and related spatio-temporal phenomena, especially those connected to "disordered" materials

CONCLUSIONS / FUTURISTIC EXTENSIONS

The above applications of chaos are in addition to the applications of chaos in telecommunications, specifically utilizing the properties of periodicity embedded within intermittency or bursting behavior towards channel coding (especially utilizing the mixing properties to reduce burst errors) and encryption and utilizing the sensitivity to .initial conditions for increasing the degree of multiplexing.

It would be interesting to explore the idea of utilizing the AO cell in phased array radar (or rather lidar) based applications where a scanning beam is generated through electronic means without involving moving parts. Interestingly, the ideas of agile beam steering, beam forming, directional chaos, control of chaos etc that are involved in the above applications could also possibly be used to improve directionality of hearing aids , microphones etc with minimal footprint. (For example it would be interesting to consider the effects of light being used to scatter acoustic waves (which is exactly the reverse of the normal functioning of an AO cell), especially those with wave numbers that are to be jammed).

Other applications include the idea of harnessing the above ideas towards tracking (and possibly destroying or slowing down objects such as asteroids / meteorites (meteorites and parachutes !!!!!) from outer space with irregular / chaotic or otherwise evasive trajectories. (This also seems to be indirectly connected to the idea of tracking prospective fraudsters in commercial transactions!!!)

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